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1996 J. Phys.: Condens. Matter 8 457

(http://iopscience.iop.org/0953-8984/8/4/011)

J. Phys.: Condens. Matter **8** (1996) 457–472. Printed in the UK

Landau Fermi-liquid theory for heavy-fermion compounds: II. Collective modes

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Received 4 July 1995, in final form 15 September 1995

Abstract. We generalize the Landau Fermi-liquid theory for describing kinetic phenomena in heavy-fermion compounds at low temperatures. The kinetic equation is derived and solved for the case when collisions between quasi-particles are negligible. Collective zero-sound and spin-wave modes are studied. We found that zero-sound modes are broken down by phase fluctuations. Symmetric collective spin excitations consist of acoustic-like and optical-like modes. The former excitations are decaying, whereas the latter excitations are stable. Electron spin resonance is also investigated. It is found that localized f electrons give the main contribution to the resonance and determine the resonance frequency. The usual Landau parameters F_0^s and F_0^a for heavy-fermion systems are determined.

1. Introduction

Not only the thermodynamic properties but also the kinetic phenomena in heavy-fermion compounds are unusual [1–3]. The specific properties of these compounds are determined by strong interactions between conduction electrons and localized electrons of partially filled f shells of rare-earth and uranium ions. It is well known that with decreasing temperature in the compounds a gradual transition from incoherent Kondo scattering to coherent scattering takes place. It produces a marked effect on almost all the properties of the compounds. At temperatures below the Kondo temperature T_0 the coherent Kondo state is formed. In this state the compounds behave as normal Fermi liquids with heavy quasi-particles near the Fermi surface and a very low Fermi temperature of the order of T_0. Evidence for this may be found in the temperature dependences of the heat capacity, magnetic susceptibility and conductivity [1–3]. Some peculiarities of kinetic phenomena in the compounds may be described within slave-boson models via the $1/N$ expansion [4–9]. This approach also enables us to calculate some dynamic correlation functions [6, 9, 10]. Unfortunately the application of the $1/N$ expansion to conventional models encounters important problems. One of them is related to taking into account the RKKY interaction between f electrons. The next problem is related to the time description of the dynamic phenomena. Calculations of dynamic correlation functions within high-degeneracy models are based on the Mazubara method within which we have to deal with imaginary time and frequencies with a subsequent analytical continuation on the real axis.

In the present paper we derive a description of dynamic phenomena in heavy-fermion compounds within the generalized Landau Fermi-liquid theory proposed in our previous paper [11] where we have shown that the phenomenological approach gives a detailed description of the low-temperature thermodynamic properties of the heavy-fermion ground

0953-8984/96/040457+16$19.50

state in agreement with microscopic approaches [8, 9]. Moreover the Landau-like approach has enabled us to study the ferromagnetic phase transition stimulated by the RKKY interaction. In the present paper we shall consider dynamic phenomena that arise for a small deviation from the equilibrium heavy-fermion state. In section 2 the linearized kinetic equation is derived and solved in the case when collisions between quasi-particles may be neglected. Then in section 3 we investigate zero-sound modes. We shall show that the modes are completely suppressed by phase fluctuations that are specific to the heavy-fermion system in which electrons form a coherent Fermi liquid. In section 4, collective spin-wave excitations are studied. It should be noted that in this case the RKKY interaction plays a very important role. We shall show that the spectrum of symmetric spin-wave excitations consists of acoustic-like and optical-like spin excitations. The former excitations are decaying, but the latter excitations are stable. In section 5 we shall study electron spin resonance. Finally, section 6 gives a discussion of our results.

2. Kinetic equation

Let us consider an electron system that consists of conduction electrons in a wide conduction band $\varepsilon(\boldsymbol{k})$ and electrons in a very narrow f band. If the dispersion of the f band is negligible, then f electrons may be supposed to be localized on the energy level ε_{f} that is placed sufficiently deeply under the Fermi surface. If the Hubbard repulsion between f electrons on the level is large enough, then the number of electrons on the level is fixed and is about 1. An interaction between conduction electrons and f electrons is caused by electron transitions between states in the conduction and f bands. We also suppose that the crystal-field interactions leave us with a double ground state.

In accordance with the generalized Landau Fermi-liquid theory proposed in [11] an equilibrium distribution of the conduction and f electrons in the system is described by a distribution matrix $N^{ab}_{0\alpha\beta}(\boldsymbol{k})$ with the band indices $a, b = c, f$ and spin indices α and β for $s = \frac{1}{2}$ spins. It is convenient to present the matrix in the block form

$$N_0(\boldsymbol{k}) = \begin{bmatrix} N^{c}_{0\alpha\beta}(\boldsymbol{k}) & N^{cf}_{0\alpha\beta}(\boldsymbol{k}) \\ N^{fc}_{0\alpha\beta}(\boldsymbol{k}) & N^{f}_{0\alpha\beta}(\boldsymbol{k}) \end{bmatrix}. \tag{2.1}$$

The diagonal elements $N^{c}_{0\alpha\beta}(\boldsymbol{k})$ and $N^{f}_{0\alpha\beta}(\boldsymbol{k})$ describe the equilibrium distribution of electrons over states in the conduction and f bands, respectively. The non-diagonal elements $(N^{cf}_{0\alpha\beta}(\boldsymbol{k}) = (N^{fc}_{0\beta\alpha}(\boldsymbol{k}))^*)$ arise owing to the assumption that, in the low-temperature region $T \ll T_0$, quasi-particle states near the Fermi surface are a quantum superposition of electron states in the conduction and f bands. Unlike the diagonal matrix elements that are real functions, in the general case the non-diagonal elements are complex functions.

Let us study the fluctuations of the distribution matrix about the equilibrium matrix N_0

$$N(\boldsymbol{k}, \boldsymbol{r}, t) = N_0(\boldsymbol{k}) + N_1(\boldsymbol{k}, \boldsymbol{r}, t). \tag{2.2}$$

The fluctuations lead to space and time dependences of the quasi-particle energy matrix

$$\varepsilon(\boldsymbol{k}, \boldsymbol{r}, t) = \varepsilon_0(\boldsymbol{k}) + \varepsilon_1(\boldsymbol{k}, \boldsymbol{r}, t). \tag{2.3}$$

If we take into account only the exchange interaction between conduction and f electrons and neglect the potential interactions between electrons, then, in the equilibrium state, elements of the quasi-particle energy matrix are given by

$$\begin{aligned}
\varepsilon^{c}_{0\alpha\beta}(\boldsymbol{k}) &= \varepsilon(\boldsymbol{k})\delta_{\alpha\beta} - \boldsymbol{h}^{c}_{0}(\boldsymbol{k})\cdot\boldsymbol{\sigma}_{\alpha\beta} \\
\varepsilon^{f}_{0\alpha\beta}(\boldsymbol{k}) &= \tilde{\varepsilon}_{f}\delta_{\alpha\beta} - \boldsymbol{h}^{f}_{0}(\boldsymbol{k})\cdot\boldsymbol{\sigma}_{\alpha\beta} \\
\varepsilon^{cf}_{0\alpha\beta}(\boldsymbol{k}) &= b(\boldsymbol{k})\delta_{\alpha\beta}
\end{aligned} \tag{2.4}$$

where $\boldsymbol{h}^{c}_{0}(\boldsymbol{k})$ and $\boldsymbol{h}^{f}_{0}(\boldsymbol{k})$ are the equilibrium effective magnetic fields which affect conduction and f electrons, respectively [11]. These fields and the function $b(\boldsymbol{k})$ that characterizes the formation of the coherent state are related to the equilibrium distribution matrix N_0:

$$\begin{aligned}
\boldsymbol{h}^{c}_{0}(\boldsymbol{k}) &= \tfrac{1}{2}g_c\mu_B\boldsymbol{H}_0 - \frac{1}{N_u}\sum_{\gamma\delta\boldsymbol{p}} G(\boldsymbol{k},\boldsymbol{p})\boldsymbol{\sigma}_{\gamma\delta}N^{f}_{0\delta\gamma}(\boldsymbol{p}) \\
\boldsymbol{h}^{f}_{0}(\boldsymbol{k}) &= \tfrac{1}{2}g_f\mu_B\boldsymbol{H}_0 - \frac{1}{N_u}\sum_{\gamma\delta\boldsymbol{p}} G(\boldsymbol{k},\boldsymbol{p})\boldsymbol{\sigma}_{\gamma\delta}N^{c}_{0\delta\gamma}(\boldsymbol{p}) \\
b(\boldsymbol{k}) &= \frac{1}{N_u}\sum_{\gamma\boldsymbol{p}} \varphi(\boldsymbol{k},\boldsymbol{p})N^{cf}_{0\gamma\gamma}(\boldsymbol{p})
\end{aligned} \tag{2.5}$$

where N_u is the number of unit cells. Below we shall suppose that an external magnetic field $\boldsymbol{H}_0$ is parallel to the z axis. The functions $G(\boldsymbol{k},\boldsymbol{p})$ and $\varphi(\boldsymbol{k},\boldsymbol{p})$ describe the exchange interaction between conduction electrons in the band $\varepsilon(\boldsymbol{k})$ and f electrons on the f level with the effective energy $\tilde{\varepsilon}_f$. In the general case the coherence parameter $b(\boldsymbol{k})$ is complex. Hence, it is characterized by the magnitude and phase. Phase fluctuations are specific for heavy-fermion compounds, but their role has been poorly studied as yet.

As we take into account no potential interaction, fluctuations (ε_1) of the local quasi-particle energy occur due to the fluctuations of local effective fields $\boldsymbol{h}^{c}_{1}$ and $\boldsymbol{h}^{f}_{1}$ acting on conduction and f electrons, respectively, and due to fluctuations of the effective f-level energy:

$$\begin{aligned}
\varepsilon^{c}_{1\alpha\beta}(\boldsymbol{k},\boldsymbol{r},t) &= -\boldsymbol{h}^{c}_{1}(\boldsymbol{k},\boldsymbol{r},t)\boldsymbol{\sigma}_{\alpha\beta} \\
\varepsilon^{f}_{1\alpha\beta}(\boldsymbol{k},\boldsymbol{r},t) &= \varepsilon_{1f}(\boldsymbol{r},t)\delta_{\alpha\beta} - \boldsymbol{h}^{f}_{1}(\boldsymbol{k},\boldsymbol{r},t)\boldsymbol{\sigma}_{\alpha\beta}.
\end{aligned} \tag{2.6}$$

The fields $\boldsymbol{h}^{c}_{1}$ and $\boldsymbol{h}^{f}_{1}$ are related to the matrix N_1:

$$\boldsymbol{h}^{c}_{1}(\boldsymbol{k},\boldsymbol{r},t) = \tfrac{1}{2}g_c\mu_B\boldsymbol{H}_1(\boldsymbol{r},t) - \frac{1}{N_u}\sum_{\gamma\delta\boldsymbol{p}} G(\boldsymbol{k},\boldsymbol{p})\boldsymbol{\sigma}_{\gamma\delta}N^{f}_{1\delta\gamma}(\boldsymbol{p},\boldsymbol{r},t) \tag{2.7a}$$

$$\boldsymbol{h}^{f}_{1}(\boldsymbol{k},\boldsymbol{r},t) = \tfrac{1}{2}g_f\mu_B\boldsymbol{H}_1(\boldsymbol{r},t) - \frac{1}{N_u}\sum_{\gamma\delta\boldsymbol{p}} G(\boldsymbol{k},\boldsymbol{p})\boldsymbol{\sigma}_{\gamma\delta}N^{c}_{1\delta\gamma}(\boldsymbol{p},\boldsymbol{r},t). \tag{2.7b}$$

Here $\boldsymbol{H}_1(\boldsymbol{r},t)$ is the time- and space-dependent part of the external magnetic field. At the same time the fluctuations N_1 in the distribution matrix lead to fluctuations in the coherence parameter:

$$\varepsilon^{cf}_{1\alpha\beta}(\boldsymbol{k},\boldsymbol{r},t) = (\varepsilon^{fc}_{1\beta\alpha}(\boldsymbol{k},\boldsymbol{r},t))^{*} = \delta_{\alpha\beta}N_u^{-1}\sum_{\gamma\boldsymbol{p}}\varphi(\boldsymbol{k},\boldsymbol{p})N^{cf}_{1\gamma\gamma}((\boldsymbol{p},\boldsymbol{r},t). \tag{2.7c}$$

It is important to note that in the general case the local effective energy of f electrons ($\varepsilon_f(\boldsymbol{r},t) = \tilde{\varepsilon}_f + \varepsilon^{f}_{1}(\boldsymbol{r},t)$) depends on $\boldsymbol{r}$ and t. In accordance with the Landau-like approach to heavy fermion systems [11] the value of $\varepsilon_f(\boldsymbol{r},t)$ is determined by the constraint that the number N_f of f electrons does not depend on $\boldsymbol{r}$ and t in low-frequency processes, i.e.

$$N_f = \sum_{\boldsymbol{k}\alpha} N^{f}_{\alpha\alpha}(\boldsymbol{k},\boldsymbol{r},t) = \text{constant}. \tag{2.8}$$

The equilibrium value of the effective energy $\tilde{\varepsilon}_f$ is determined by

$$N_f = \sum_{\boldsymbol{k},\alpha} N^f_{0\alpha\alpha}(\boldsymbol{k}, \boldsymbol{r}, t). \tag{2.9}$$

Hence, taking into account (2.2), we can write the constraint (2.8) for the fluctuations in the form

$$\sum_{\boldsymbol{k},\alpha} N^f_{1\alpha\alpha}(\boldsymbol{k}, \boldsymbol{r}, t) = 0. \tag{2.10}$$

From the physical point of view this constraint is related to the fact that a change in the occupancy of f level requires too large an energy. Therefore, fluctuations in the number of f electrons are sufficiently small and negligible in the low-frequency processes that only will be considered in the present paper.

In accordance with the general principles of the Landau Fermi-liquid theory (see, e.g., [12, 13]) the time and space dependences of the distribution matrix are determined by the kinetic equation

$$\frac{\partial N}{\partial t} + \{\nabla_{\boldsymbol{r}} N \nabla_{\boldsymbol{k}} \varepsilon\} - \{\nabla_{\boldsymbol{k}} N \nabla_{\boldsymbol{r}} \varepsilon\} - \mathrm{i}[\varepsilon, N] = I(N_1) \tag{2.11}$$

where the curly brackets mean that $2\{AB\} = AB + BA$. The collision integral $I(N_1)$ describes a change in the distribution matrix owing to electron collisions. Summing over $\boldsymbol{k}$ and taking the trace over the band and spin indices in this equation lead to the continuity equation

$$\partial n/\partial t + \operatorname{div} \boldsymbol{J} = 0$$

where n is the total electron concentration:

$$n = N_u^{-1} \sum_{\boldsymbol{k}\alpha} (N^c_{\alpha\alpha}(\boldsymbol{k}, \boldsymbol{r}, t) + N^f_{\alpha\alpha}(\boldsymbol{k}, \boldsymbol{r}, t))$$

and $\boldsymbol{J}$ is the quasi-particle flow. Using (2.4) and (2.6), for the paramagnetic state we find that

$$\boldsymbol{J} = N_u^{-1} \sum_{\boldsymbol{k}\alpha} (\boldsymbol{v}_{0\boldsymbol{k}} N^c_{\alpha\alpha}(\boldsymbol{k}, \boldsymbol{r}, t) + N^{cf}_{\alpha\alpha}(\boldsymbol{k}, \boldsymbol{r}, t) \nabla_{\boldsymbol{k}} \varepsilon^{fc}_{\alpha\alpha}(\boldsymbol{k}, \boldsymbol{r}, t) + N^{fc}_{\alpha\alpha}(\boldsymbol{k}, \boldsymbol{r}, t) \nabla_{\boldsymbol{k}} \varepsilon^{cf}_{\alpha\alpha}(\boldsymbol{k}, \boldsymbol{r}, t))$$

where $\boldsymbol{v}_{0\boldsymbol{k}} = \nabla_{\boldsymbol{k}} \varepsilon(\boldsymbol{k})$ is the non-renormalized velocity of conduction electrons. The first term in this equation is the contribution of conduction electrons to the quasi-particle flow. The physical meaning of the other terms is not yet clear. If the function $\varphi(\boldsymbol{k}, \boldsymbol{p})$ is constant, then according to (2.7c) the coherence parameter ε^{cf} does not depend on $\boldsymbol{k}$, i.e. $\nabla_{\boldsymbol{k}} \varepsilon^{fc}_{\alpha\alpha} = 0$. Then $\boldsymbol{J}$ takes the physically clear form

$$\boldsymbol{J} = N_u^{-1} \sum_{\boldsymbol{k}\alpha} \boldsymbol{v}_{0\boldsymbol{k}} N^c_{\alpha\alpha}(\boldsymbol{k}, \boldsymbol{r}, t)$$

which shows that only conduction electrons contribute to the flow.

In the present paper we shall neglect the collisions. This means that we can only consider processes with a frequency ω larger than the frequency of the collisions. For pure samples at low temperatures $T \ll T_0$ the collision frequency may be sufficiently small. In this case the linearized kinetic equation (2.11) takes the form

$$\partial N_1/\partial t + \{\nabla_{\boldsymbol{r}} N_1 \nabla_{\boldsymbol{k}} \varepsilon_0\} - \{\nabla_{\boldsymbol{k}} N_0 \nabla_{\boldsymbol{r}} \varepsilon_1\} - \mathrm{i}[\varepsilon_1, N_0] - \mathrm{i}[\varepsilon_0, N_1] = 0 \tag{2.12}$$

where ε_0 and N_0 are the equilibrium quasi-particle energy matrix (2.4) and distribution matrix (2.1).

Let us solve equation (2.12) when the characteristic space scale of the fluctuations is much larger than the lattice constant. This means that the Fourier transformation

$$N_1(\boldsymbol{k},\boldsymbol{r},t)=\sum_{\boldsymbol{q}}\exp(\mathrm{i}\boldsymbol{q}\cdot\boldsymbol{r})N_1(\boldsymbol{k},\boldsymbol{q},t)$$

contains only wavenumbers $q \ll k_F$. After the Fourier transformation the linearized kinetic equation (2.12) looks like

$$\partial N_1/\partial t+\mathrm{i}\{N_1(\boldsymbol{q}\nabla_{\boldsymbol{k}}\varepsilon_0)\}-\mathrm{i}\{(\boldsymbol{q}\nabla_{\boldsymbol{k}}N_0)\varepsilon_1\}-\mathrm{i}[\varepsilon_1,N_0]-\mathrm{i}[\varepsilon_0,N_1]=0 \quad (2.13)$$

where $N_1 \equiv N_1(\boldsymbol{k},\boldsymbol{r},t)$ and $\varepsilon_1 \equiv \varepsilon_1(\boldsymbol{k},\boldsymbol{r},t)$. To solve this equation, it is convenient to rewrite it in the form

$$\mathrm{i}\frac{\partial}{\partial t}N_1(\boldsymbol{k},\boldsymbol{r},t)=N_1(\boldsymbol{k},\boldsymbol{r},t)\varepsilon_+-\varepsilon_-N_1(\boldsymbol{k},\boldsymbol{q},t)+N_-\varepsilon_1(\boldsymbol{k},\boldsymbol{q},t)-\varepsilon_1(\boldsymbol{k},\boldsymbol{q},t)N_+ \quad (2.14)$$

where for brevity we denote

$$\varepsilon_\pm=\varepsilon_0(\boldsymbol{k})\pm\tfrac{1}{2}\boldsymbol{q}\nabla_{\boldsymbol{k}}\varepsilon_0(\boldsymbol{k}) \qquad N_\pm=N_0(\boldsymbol{k})\pm\tfrac{1}{2}\boldsymbol{q}\nabla_{\boldsymbol{k}}N_0(\boldsymbol{k}). \quad (2.15)$$

After the transformation

$$N_1=\exp(\mathrm{i}\varepsilon_-t)\tilde{N}\exp(-\mathrm{i}\varepsilon_+t) \quad (2.16)$$

equation (2.14) looks like

$$\frac{1}{\mathrm{i}}\frac{\partial}{\partial t}\tilde{N}_1=\exp(-\mathrm{i}\varepsilon_-t)(\varepsilon_1N_+-N_-\varepsilon_1)\exp(\mathrm{i}\varepsilon_+t). \quad (2.17)$$

This enables us to find $\tilde{N}_1$ and then N_1. So the general solution of the linearized kinetic equation (2.13) takes the form

$$N_1(\boldsymbol{k},\boldsymbol{q},t)=\mathrm{i}\int_{-\infty}^{0}\exp(-\mathrm{i}\varepsilon_-y)(\varepsilon_1(\boldsymbol{k},\boldsymbol{q},t+y)N_+-N_-\varepsilon_1(\boldsymbol{k},\boldsymbol{q},t+y))\exp(\mathrm{i}\varepsilon_+y)\,\mathrm{d}y. \quad (2.18)$$

However, this solution is too complicated. In many cases, one can use an approximate solution that can be found in the following way. One can note that the equilibrium matrices $\varepsilon_0(\boldsymbol{k}\pm\boldsymbol{q}/2)$ and $N_0(\boldsymbol{k}\pm\boldsymbol{q}/2)$ differ from matrices (2.15) by terms $\mathrm{O}(q^2)$. At small q when the terms may be neglected, one can use the following substitution in equation (2.18):

$$\varepsilon_\pm=\varepsilon_0(\boldsymbol{k}\pm\boldsymbol{q}/2) \qquad N_\pm=N_0(\boldsymbol{k}\pm\boldsymbol{q}/2). \quad (2.19)$$

As in the equilibrium state the matrices $\varepsilon_0(\boldsymbol{k})$ and $N_0(\boldsymbol{k})$ commutate with each other at arbitrary $\boldsymbol{k}$, one can diagonalize them simultaneously, using a unitary transformation $U_{\boldsymbol{k}}$:

$$\varepsilon_0(\boldsymbol{k})=U_{\boldsymbol{k}}^{-1}E_{\boldsymbol{k}}U_{\boldsymbol{k}} \qquad N_0(\boldsymbol{k})=U_{\boldsymbol{k}}^{-1}f_{\boldsymbol{k}}U_{\boldsymbol{k}} \quad (2.20)$$

where

$$E_{\boldsymbol{k}}=\begin{bmatrix} E_{1\boldsymbol{k}\alpha}\delta_{\alpha\beta} & 0\\ 0 & E_{2\boldsymbol{k}\alpha}\delta_{\alpha\beta}\end{bmatrix} \quad (2.21)$$

$$f_{\boldsymbol{k}}=\begin{bmatrix} f(E_{1\boldsymbol{k}\alpha})\delta_{\alpha\beta} & 0\\ 0 & f(E_{2\boldsymbol{k}\alpha})\delta_{\alpha\beta}\end{bmatrix}. \quad (2.22)$$

Here $E_{1\boldsymbol{k}\alpha}$ and $E_{2\boldsymbol{k}\alpha}$ are quasi-particle energies in two hybridized bands that will be determined below. Moreover $f(x)=[\exp((x-\mu)/T)+1]^{-1}$. Inserting (2.19)–(2.22) into (2.18) gives the approximate solution for the fluctuation matrix N_1:

$$N_1(\boldsymbol{k},\boldsymbol{q},t)=\mathrm{i}\int_{-\infty}^{0}\mathrm{d}y\,U_-^{-1}\exp(-\mathrm{i}E_-y)(U_-\varepsilon_1(\boldsymbol{k},\boldsymbol{q},t+y)U_+^{-1}f_+$$
$$-f_-U_-\varepsilon_1(\boldsymbol{k},\boldsymbol{q},t+y)U_+^{-1})\exp(\mathrm{i}E_+y)U_+. \quad (2.23)$$

This approximate solution of the kinetic equation (2.13) will be used below for studying collective excitations.

3. Zero-sound modes

Now we shall study long-length zero-sound modes in the neutral heavy-fermion system. Zero-sound oscillations change the electron density and do not change the spin density. In zero magnetic field the oscillations are described by fluctuations of the distribution matrix $N^{ab}_{1\alpha\beta} = N^{ab}_1\delta_{\alpha\beta}$ and the quasi-partial energy matrix $\varepsilon^{ab}_{1\alpha\beta} = \varepsilon^{ab}_1\delta_{\alpha\beta}$. Then, in accordance with (2.6) and (2.7) in the $\boldsymbol{q}$ representation, the matrix elements ε^{ab}_1 have the form

$$\varepsilon^c_1 = 0 \qquad \varepsilon^f_1 = \varepsilon^f_1(\boldsymbol{q}, t) \tag{3.1a}$$

$$\varepsilon^{cf}_1(\boldsymbol{k}, \boldsymbol{q}, t) = (\varepsilon^{fc}_1(\boldsymbol{k}, -\boldsymbol{q}, t))^* = 2N_u^{-1}\sum_{\gamma\boldsymbol{p}}\varphi(\boldsymbol{k}, \boldsymbol{p})N^{cf}_1(\boldsymbol{p}, \boldsymbol{q}, t). \tag{3.1b}$$

Note that ε^c_1 is equal to zero because we neglect potential interactions between electrons. Thus zero-sound oscillations are described by three unknown functions: $\varepsilon^f_1(\boldsymbol{q}, t)$, $\varepsilon^{cf}_1(\boldsymbol{k}, \boldsymbol{q}, t)$ and $\varepsilon^{cf}_1(\boldsymbol{k}, \boldsymbol{q}, t)$. For simplicity we shall consider only the case of the isotropic Fermi surface. At zero magnetic field the equilibrium matrices N_0 and ε_0 have the simple spin structure $N^{ab}_{0\alpha\beta} = N^{ab}_0\delta_{\alpha\beta}$ and $\varepsilon^{ab}_{0\alpha\beta} = \varepsilon^{ab}_0\delta_{\alpha\beta}$. According to (2.4) and (2.5), in equilibrium the elements of the quasi-particle energy matrix are equal to

$$\varepsilon^c_0(\boldsymbol{k}) = \varepsilon(\boldsymbol{k}) \qquad \varepsilon^f_0(\boldsymbol{k}) = \tilde{\varepsilon}_f \qquad \varepsilon^{cf}_0(\boldsymbol{k}) = b \tag{3.2}$$

where the coherence parameter b given by (2.5) does not depend on $\boldsymbol{k}$. The unitary transformation $U_{\boldsymbol{k}}$ in (2.20) may be written as

$$U_{\boldsymbol{k}} = \begin{pmatrix} \cos\theta_{\boldsymbol{k}} & -\sin\theta_{\boldsymbol{k}} \\ \sin\theta_{\boldsymbol{k}} & \cos\theta_{\boldsymbol{k}} \end{pmatrix}. \tag{3.3}$$

Then there are the following relations between the quasi-particle energies $E_{1\boldsymbol{k}}$, $E_{2\boldsymbol{k}}$ and the angle $\theta_{\boldsymbol{k}}$:

$$\begin{aligned} E_{1\boldsymbol{k}} &= \tilde{\varepsilon}_f - b\cot\theta_{\boldsymbol{k}} = \tfrac{1}{2}(\varepsilon(\boldsymbol{k}) + \tilde{\varepsilon}_f - [(\varepsilon(\boldsymbol{k}) - \tilde{\varepsilon}_f)^2 + 4b^2]^{1/2}) \\ E_{2\boldsymbol{k}} &= \tilde{\varepsilon}_f + b\tan\theta_{\boldsymbol{k}} = \tfrac{1}{2}(\varepsilon(\boldsymbol{k}) + \tilde{\varepsilon}_f + [(\varepsilon(\boldsymbol{k}) - \tilde{\varepsilon}_f)^2 + 4b^2]^{1/2}). \end{aligned} \tag{3.4}$$

It is easy to make sure that there is a gap of the order of the low-temperature Kondo scale T_0 between the lower band $E_{1\boldsymbol{k}}$ and the upper band $E_{2\boldsymbol{k}}$. Below we shall suppose that the total number ($N_t = N_c + N_f$) of conduction and f electrons is less than 2. In this case at zero temperature the lower band is partially filled whereas the upper band is empty, i.e. $f(E_{2\boldsymbol{k}}) = 0$.

Let us consider symmetric zero-sound modes. In this case, three unknown functions $\varepsilon^{ab}_1(\boldsymbol{k}, \boldsymbol{q}, t)$ in (3.1) do not depend on $\boldsymbol{k}$. We shall look for them in a time periodic form

$$\varepsilon^{ab}_1(\boldsymbol{k}, \boldsymbol{q}, t) = \varepsilon^{ab}_1(\boldsymbol{q}, \omega)\exp(\mathrm{i}\omega t). \tag{3.5}$$

In accord with (3.1) we have $\varepsilon^{cf}_1(\boldsymbol{q}, \omega) = (\varepsilon^{fc}_1(-\boldsymbol{q}, -\omega))^*$. Substituting the matrix ε_1 given by (3.1) and (3.5) into equation (2.23), after simple but bulky calculations at $q \ll k_F$, one obtains the fluctuation matrix N_1. The result of the calculation is presented in appendix 1.

In order to determine three unknown parameters ε^f_1, ε^{cf}_1 and ε^{fc}_1 we must solve self-consistently equations (2.10) and (3.1b). Substituting equations (A1.2) and (A1.3) from appendix 1 into these equations gives a set of linear algebraic equations:

$$\begin{aligned}
&\varepsilon_1^f(\sin^2\theta_F\tan^2\theta_F\Gamma(q,\omega)-A(\omega))+\varepsilon_1^{cf}(-\sin^2\theta_F\tan\theta_F\Gamma(q,\omega)+B(\omega))\\
&\qquad+\varepsilon_1^{fc}(-\sin^2\theta_F\tan\theta_F\Gamma(q,\omega)+B(-\omega))=0\\
&\varepsilon_1^f(-\sin^2\theta_F\tan\theta_F\Gamma(q,\omega)+B(\omega))+\varepsilon_1^{cf}(\sin^2\theta_F\Gamma(q,\omega)-C(\omega))\\
&\qquad+\varepsilon_1^{fc}(\sin^2\theta_F\Gamma(q,\omega)+A(\omega))=\varepsilon_1^{cf}/2\varphi_0\rho_0\\
&\varepsilon_1^f(-\sin^2\theta_F\tan\theta_F\Gamma(q,\omega)+B(-\omega))+\varepsilon_1^{cf}(\sin^2\theta_F\Gamma(q,\omega)+A(\omega))\\
&\qquad+\varepsilon_1^{fc}(\sin^2\theta_F\Gamma(q,\omega)-C(-\omega))=\varepsilon_1^{fc}/2\varphi_0\rho_0.
\end{aligned}\tag{3.6}$$

Here we introduce the following functions:

$$A(\omega)=(2\rho_0N_u)^{-1}\sum_{\boldsymbol{k}}f(E_{1\boldsymbol{k}})\Delta_{\boldsymbol{k}}\sin^2(2\theta_{\boldsymbol{k}})(\Delta_{\boldsymbol{k}}^2-\omega^2)^{-1}\tag{3.7}$$

$$B(\omega)=(2\rho_0N_u)^{-1}\sum_{\boldsymbol{k}}f(E_{1\boldsymbol{k}})\sin(2\theta_{\boldsymbol{k}})\left(\frac{\cos^2\theta_{\boldsymbol{k}}}{\Delta_{\boldsymbol{k}}-\omega}-\frac{\sin^2\theta_{\boldsymbol{k}}}{\Delta_{\boldsymbol{k}}+\omega}\right)\tag{3.8}$$

$$C(\omega)=(\rho_0N_u)^{-1}\sum_{\boldsymbol{k}}f(E_{1\boldsymbol{k}})\left(\frac{\cos^4\theta_{\boldsymbol{k}}}{\Delta_{\boldsymbol{k}}-\omega}+\frac{\sin^4\theta_{\boldsymbol{k}}}{\Delta_{\boldsymbol{k}}+\omega}\right)\tag{3.9}$$

$$\Gamma(q,\omega)=(\rho_0N_u)^{-1}\sum_{\boldsymbol{k}}\cdot\frac{(\boldsymbol{q}\cdot\boldsymbol{v}_{\boldsymbol{k}})\cos^2\theta_{\boldsymbol{k}}}{(\boldsymbol{q}\cdot\boldsymbol{v}_{\boldsymbol{k}})-\omega}f'(E_{1\boldsymbol{k}})=\frac{\lambda}{2}\ln\left|\frac{\lambda+1}{\lambda-1}\right|-1\tag{3.10}$$

where $\Delta_{\boldsymbol{k}}=E_{2\boldsymbol{k}}-E_{1\boldsymbol{k}}$ is the direct energy gap between the bands (3.4), $\lambda\equiv\omega/qv_F$ and φ_0 is the term with $l=0$ in the expansion of the interaction function $\varphi(\boldsymbol{k},\boldsymbol{p})$ in the Legendre polynomials. According to [11], for the heavy-fermion system the parameter φ_0 must be negative. ρ_0 is the density of states in the conduction band $\varepsilon(\boldsymbol{k})$ near the Fermi surface.

For analysing the set of equations it is suitable to use the following relations between the functions $A(\omega)$, $B(\omega)$ and $C(\omega)$:

$$\begin{aligned}
B(\omega)&=\beta(\omega)+\frac{\omega}{2b}A(\omega)\\
C(\omega)&=\frac{1}{2|\varphi_0|\rho_0}+(\tfrac{1}{2}\omega^2b^{-2}-1)A(\omega)+\frac{\omega}{b}\beta(\omega)
\end{aligned}\tag{3.11}$$

where we define $\beta(\omega)=(B(\omega)+B(-\omega))/2$. The latter equation in (3.11) follows from equation (3.9) and the equality

$$\frac{1}{2|\varphi_0|}=N_u^{-1}\sum_{k<k_F}\Delta_{\boldsymbol{k}}^{-1}\tag{3.12}$$

(see equation (3.9) in [11] at $T=0$). Let us introduce new variables whose physical meaning will be discussed below:

$$\begin{aligned}
\varepsilon_1'&=\varepsilon_1^{cf}+\varepsilon_1^{fc}\\
\varepsilon_1''&=\varepsilon_1^{cf}-\varepsilon_1^{fc}.
\end{aligned}\tag{3.13}$$

Summing and subtracting the last two equations in (3.6) gives

$$\varepsilon_1^f(\sin^2\theta_F\tan^2\theta_F\Gamma(q,\omega)-A(\omega))+\varepsilon_1'(-\sin^2\theta_F\tan\theta_F\Gamma(q,\omega)+\beta(\omega))+\frac{\omega}{2b}A(\omega)\varepsilon_1''=0\tag{3.14a}$$

$$\varepsilon_1^f(-\sin^2\theta_F \tan\theta_F \Gamma(q,\omega) + \beta(\omega)) + \varepsilon_1'(\sin^2\theta_F \Gamma(q,\omega) + (1 - \tfrac{1}{4}\omega^2 b^{-2})A(\omega)) - \frac{\omega}{2b}A(\omega)\varepsilon_1'' = 0 \qquad (3.14b)$$

$$\frac{\omega}{2b}(\varepsilon_1^f A(\omega) - \varepsilon_1'\beta(\omega) - \frac{\omega}{2b}A(\omega)\varepsilon_1'') = 0. \qquad (3.14c)$$

Now this set of equations may be easily solved. Equation (3.14*c*) gives $\omega\varepsilon_1''/2b = \varepsilon_1^f - \varepsilon_1'\beta(\omega)/A(\omega)$. Then equation (3.14*a*) leads to $\varepsilon_1' = \varepsilon_1^f \tan\theta_F$. Finally, equation (3.14*b*) has a non-trivial solution if

$$(1 - \tfrac{1}{4}\omega^2 b^{-2})A^2(\omega) + \beta^2(\omega) = 0. \qquad (3.15)$$

As $\min \Delta_{\boldsymbol{k}} = 2b$, the functions $A(\omega)$ and $\beta(\omega)$ given by (3.7), (3.8) and (3.11) are real analytical functions of ω in the range $-2b < \omega < 2b$. Moreover, we have $A(\omega) > 0$. Hence, the right-hand side of equation (3.15) is positive and the equation has no real solution in the frequency range. In the complex plane the functions have cuts along the real axis at $|\omega| > 2b$, because the integrands in $\boldsymbol{k}$ have poles at $|\omega| = \Delta_{\boldsymbol{k}}$ that are related to resonance with the interband electron transitions. Thus we conclude that only a complex solution of (3.15) is possible. This means that there are no stable zero modes in the neutral heavy-fermion system. This result contradicts the microscopic slave-boson approach [6]. To understand the origin of the contradiction we consider the low-frequency region $\omega \ll T_0 \ll b$ and neglect the terms of order ω/b in (3.14). Within this approximation, fluctuations in the parameter ε_1'' are decoupled from fluctuations in ε^f and ε'. In the frequency range the functions $A(\omega)$ and $\beta(\omega)$ in (3.14) must be replaced by their values at $\omega = 0$. Calculations of the parameters must be easily performed in the case when the band $\varepsilon(\boldsymbol{k})$ is flat. For this purpose we transform the summation over $\boldsymbol{k}$ into the integration first over the variable $E = E_{1\boldsymbol{k}}$ and then over the variable $\theta = \theta_{\boldsymbol{k}}$:

$$N_u^{-1}\sum_{\boldsymbol{k}} = \int \rho(\varepsilon)\,\mathrm{d}\varepsilon = \rho_0 \int \frac{\mathrm{d}E}{\cos^2\theta_{\boldsymbol{k}}} = 4\rho_0 b \int \frac{\mathrm{d}\theta}{\sin^2(2\theta)} \qquad (3.16)$$

where we have used the following relations that follow from (3.4):

$$\mathrm{d}E_{1\boldsymbol{k}}/\mathrm{d}\varepsilon = \cos^2\theta_{\boldsymbol{k}},\ \mathrm{d}\theta_{\boldsymbol{k}}/\mathrm{d}E_{1\boldsymbol{k}} = b^{-1}\sin^2\theta_{\boldsymbol{k}},\ \Delta_{\boldsymbol{k}} = 2b/\sin(2\theta_{\boldsymbol{k}}). \qquad (3.17)$$

Now, simple calculations give

$$\begin{aligned} A(0) &= \tfrac{1}{2}(\cos(2\theta_0) - \cos(2\theta_F)) \approx \sin^2\theta_F \\ \beta(0) &= \tfrac{1}{2}(\sin(2\theta_F) - \sin(2\theta_0)) \approx \tfrac{1}{2}\sin(2\theta_F) \end{aligned} \qquad (3.18)$$

where $\theta_0 \equiv \theta(\boldsymbol{k} = 0) \approx 0$ and $\theta_F \equiv \theta(k_F)$. Note that in equilibrium the parameters μ, $\tilde{\varepsilon}_f$ and b at zero temperature [11] are given by

$$\begin{aligned} &\tilde{\varepsilon}_f - \mu = T_0 = \mu\exp(-1/(2|\varphi_0|\rho_0)) \\ &m^*/m_0 = 1/\cos^2\theta_F = 1 + N_f/2T_0\rho_0 \gg 1 \qquad b^2 = N_f T_0/2\rho_0 \end{aligned} \qquad (3.19)$$

where m^* is the renormalized quasi-particle mass near the Fermi surface, and $\mu = N_c/2\rho_0$. Thus, within the approximation the three equations (3.14) are reduced to equations (3.14*a*) and (3.14*b*) which have a non-trivial solution if the corresponding determinant is equal to zero. So we obtain the conventional equation

$$\Gamma(q,\omega) = 1/F_0 \qquad (3.20)$$

where $F_0 = \tan^2\theta_F = m^*/m_0 - 1 \gg 1$, and $\Gamma(q,\omega)$ is given by (3.10). This equation has the solution $\omega = qv_F(m^*/3m_0)^{1/2}$. This result and the spectral equation (3.20) have been previously obtained in [6] within the slave-boson $1/N$ expansion.

The approximation considered above shows that taking into account the fluctuations of the parameter ε_1'' (3.13) is very important since the fluctuations break down the zero modes found in [6]. To understand the physical meaning of the parameter, we note that in accordance with (3.6) the parameters ε_1^f, ε_1^{cf} and ε_1^{fc} may be chosen to be simultaneously either real or imaginary. Let them be real. Hence we have

$$\varepsilon_1^{cf}(\boldsymbol{q}, \omega) = (\varepsilon_1^{fc}(-\boldsymbol{q}, -\omega))^* = \varepsilon_1^{fc}(-\boldsymbol{q}, -\omega). \tag{3.21}$$

Then, using (3.13), the local fluctuations of the coherence parameter b may be written as

$$\begin{aligned} \varepsilon_1^{cf}(\boldsymbol{r}, \omega) &= \varepsilon_1^{cf}(\boldsymbol{q}, \omega)\exp(\mathrm{i}\boldsymbol{q}\cdot\boldsymbol{r} - \mathrm{i}\omega t) + \varepsilon_1^{cf}(-\boldsymbol{q}, -\omega)\exp(-\mathrm{i}\boldsymbol{q}\cdot\boldsymbol{r} + \mathrm{i}\omega t) \\ &= \varepsilon_1'(\boldsymbol{q}, \omega)\cos(\boldsymbol{q}\cdot\boldsymbol{r} - \omega t) + \mathrm{i}\varepsilon_1''(\boldsymbol{q}, \omega)\sin(\boldsymbol{q}\cdot\boldsymbol{r} - \omega t). \end{aligned} \tag{3.22}$$

Thus, the parameters ε_1' and ε_1'' describe fluctuations in the real and imaginary parts, respectively, of the coherence parameter b (2.5). It is obvious that the phase fluctuations in the parameter are determined by ε_1''.

We conclude that it is the coherence phase fluctuations that break down the zero-mode excitations in the system considered. Equation (3.20) determining the zero-mode spectrum for normal Fermi liquids is invalid for the neutral heavy-fermion system.

It is important to note that we can identify the parameter F_0 obtained as the conventional symmetric Landau parameter F_0^s, i.e. $F_0^s = \tan^2\theta_F = m^*/m_0 - 1$. To check this result we calculate the compressibility k and find that

$$k = k_0 \frac{m^*/m_0}{1 + F_0^s} = k_0 \tag{3.23}$$

where $k_0 = 2\rho_0/N_t^2$ is related to the non-interacting system. This equation confirms the above result for F_0^s. Thus, in heavy-fermion systems at low temperatures, the compressibility tends to the unrenormalized values. Details of the calculation will be published elsewhere. Our result for F_0^s completely agrees with the microscopic theory [14] and the Gutzwiller approach [15].

4. Collective spin-wave excitations

Now we shall consider collective spin-wave excitations. Although this problem is formulated for the general case of a non-zero magnetic field, much attention will be paid only to zero-field case.

The external magnetic field $\boldsymbol{H}_0$ is directed along the axis z, and a non-uniform transverse magnetic field $\boldsymbol{H}_1(\boldsymbol{r}, t)$ is perpendicular to $\boldsymbol{H}_0$. As spin-wave excitations do not change the quasi-particle density, the condition (2.10) is fulfilled. Then, in accordance with (2.6) the matrix $\varepsilon_{1\alpha\beta}^{ab}(\boldsymbol{k}, \boldsymbol{r}, t)$ of fluctuations in the quasi-particle energy matrix about the equilibrium state (2.4) have the following elements:

$$\begin{aligned} &\varepsilon_{1\alpha\beta}^{c}(\boldsymbol{k}, \boldsymbol{r}, t) = -\boldsymbol{h}_1^c(\boldsymbol{k}, \boldsymbol{r}, t)\cdot\boldsymbol{\sigma}_{\alpha\beta} \\ &\varepsilon_{1\alpha\beta}^{f}(\boldsymbol{k}, \boldsymbol{r}, t) = -\boldsymbol{h}_1^f(\boldsymbol{k}, \boldsymbol{r}, t)\cdot\boldsymbol{\sigma}_{\alpha\beta} \\ &\varepsilon_{1\alpha\beta}^{cf}(\boldsymbol{k}, \boldsymbol{r}, t) = \varepsilon_{1\beta\alpha}^{fc}(\boldsymbol{k}, \boldsymbol{r}, t) = 0 \end{aligned} \tag{4.1}$$

and as for spin excitations, we have $\varepsilon_{1f}(\boldsymbol{r}, t) = 0$. Here the fluctuating fields $\boldsymbol{h}_1^c$ and $\boldsymbol{h}_1^f$ given by equations (2.7) are related to the fluctuation matrix N_1. The substitution of the matrix (4.1) into (2.23) enables us to find the fluctuation matrix N_1 as a function of the fluctuating fields $\boldsymbol{h}_1^c$ and $\boldsymbol{h}_1^f$. Thus equations (2.7) may be considered as integral equations for self-consistently determining the fluctuating fields. For simplicity we shall study only symmetrical spin-wave excitations. Then the fields $\boldsymbol{h}_1^c$ and $\boldsymbol{h}_1^f$ do not depend on the wavenumber $\boldsymbol{k}$.

Let us consider the time and space periodic excitations

$$\begin{aligned}\boldsymbol{h}_1^c(\boldsymbol{k},\boldsymbol{r},t) &= \boldsymbol{h}^c(\boldsymbol{q},\omega)\exp(\mathrm{i}\boldsymbol{q}\cdot\boldsymbol{r}-\mathrm{i}\omega t)+\mathrm{HC}\\ \boldsymbol{h}_1^f(\boldsymbol{k},\boldsymbol{r},t) &= \boldsymbol{h}^f(\boldsymbol{q},\omega)\exp(\mathrm{i}\boldsymbol{q}\cdot\boldsymbol{r}-\mathrm{i}\omega t)+\mathrm{HC}.\end{aligned}\tag{4.2}$$

To find the matrix N_1 from equation (2.23) it is necessary to take into account that the unitary matrix $U_{\boldsymbol{k}}$ in (2.20) has the following spin structure: $U_{\boldsymbol{k}\alpha\beta}^{ab} = U_{\boldsymbol{k}\alpha}^{ab}\delta_{\alpha\beta}$. The matrix $U_{\boldsymbol{k}\alpha}^{ab}$ looks like (3.3) with the spin-dependent parameter $\theta_{\boldsymbol{k}\alpha}$. In the external field H_0 the quasi-particle energies in the two hybridized bands have the forms:

$$\begin{aligned}E_{1\boldsymbol{k}\alpha} &= \tilde{\varepsilon}_{f\alpha} - b\cot\theta_{\boldsymbol{k}\alpha} = \tfrac{1}{2}(\varepsilon_\alpha(\boldsymbol{k})+\tilde{\varepsilon}_{f\alpha} - [(\varepsilon_\alpha(\boldsymbol{k})-\tilde{\varepsilon}_{f\alpha})^2+4b^2]^{1/2})\\ E_{2\boldsymbol{k}\alpha} &= \tilde{\varepsilon}_{f\alpha} + b\tan\theta_{\boldsymbol{k}\alpha} = \tfrac{1}{2}(\varepsilon_\alpha(\boldsymbol{k})+\tilde{\varepsilon}_{f\alpha} + [(\varepsilon_\alpha(\boldsymbol{k})-\tilde{\varepsilon}_{f\alpha})^2+4b^2]^{1/2})\end{aligned}\tag{4.3}$$

where

$$\begin{aligned}\varepsilon_\alpha(\boldsymbol{k}) &= \varepsilon(\boldsymbol{k}) - \sigma_\alpha h_0^c\\ \varepsilon_{f\alpha} &= \tilde{\varepsilon}_f - \sigma_\alpha h_0^f\end{aligned}\tag{4.4}$$

and $\sigma_\alpha = \pm 1$ for upward and downward spins, respectively; h_0^c and h_0^f are the equilibrium effective fields acting on the conduction and f electrons, respectively. According to [11], at fields $\mu_B H_0 \ll T_0$ we have

$$\begin{aligned}h_0^c &= \tfrac{1}{2}\mu_B g_c H_0(1-N_f g_f G_0/T_0 g_c)(1-T_m/T_0)^{-1}\\ h_0^f &= \tfrac{1}{2}\mu_B g_f H_0(1-T_m/T_0)^{-1}\end{aligned}\tag{4.5}$$

where G_0 is the expansion parameter with $l = 0$ of the exchange interaction function $G(\boldsymbol{k},\boldsymbol{p})$ in the Legendre polynomials. The parameter $T_m = 2N_f G_0^2\rho_0$ is the scale of the RKKY interaction between localized spins. The solution (4.5) corresponds to $T_0 > T_m$ when the system under consideration is in the paramagnetic heavy-fermion ground state. It is convenient to introduce the parameters

$$h_+^{c(f)} = h_x^{c(f)} + \mathrm{i}h_y^{c(f)}.\tag{4.6}$$

Analogously we introduce $H_{1+}(\boldsymbol{q},\omega)$ and

$$N_{1+}^{c(f)}(\boldsymbol{k},\boldsymbol{q},\omega) = \sum_{\alpha\beta}(\sigma_{\alpha\beta}^x + \mathrm{i}\sigma_{\alpha\beta}^y)N_1^{c(f)}(\boldsymbol{k},\boldsymbol{q},\omega) = 2N_{1\uparrow\downarrow}^{c(f)}(\boldsymbol{k},\boldsymbol{q},\omega).\tag{4.7}$$

Then equations (2.7) take the form

$$h_+^c(q,\omega) = \tfrac{1}{2}g_c\mu_B H_{1+}(q,\omega) - G_0 N_{u^{-1}}\sum_{\boldsymbol{p}} N_{1+}^f(\boldsymbol{p},q,\omega)\tag{4.8}$$

$$h_+^f(q,\omega) = \tfrac{1}{2}g_f\mu_B H_{1+}(q,\omega) - G_0 N_u^{-1}\sum_{\boldsymbol{p}} N_{1+}^c(\boldsymbol{p},q,\omega).\tag{4.9}$$

Substituting (4.1) into (2.23) and calculating the matrix N_1, after simple but bulky calculation we find that at $T \ll T_0$ these equations may be written as a set of algebraic equations:

$$\begin{aligned}h_+^c(q,\omega) &= \tfrac{1}{2}g_c\mu_B H_{1+}(q,\omega) + Ph_+^c(q,\omega) + Qh_+^f(q,\omega)\\ h_+^f(q,\omega) &= \tfrac{1}{2}g_f\mu_B H_{1+}(q,\omega) + Vh_+^c(q,\omega) + Rh_+^f(q,\omega)\end{aligned}\tag{4.10}$$

which determine two unknown parameters $h_+^c(q,\omega)$ and $h_+^f(q,\omega)$. The coefficients P, Q, V and R are functions of q and ω and are determined by

$$
\begin{aligned}
P = R &= \tfrac{1}{2} G_0 N_u^{-1} \sum_{p} \sin(2\theta_d)\sin(2\theta_u) \\
&\qquad \times \left(\frac{f_u - f_d}{E_{1u} - E_{1d} - \omega} - \frac{f_u}{E_{1u} - E_{2d} - \omega} + \frac{f_d}{E_{2u} - E_{1d} - \omega} \right) \\
Q &= 2G_0 N_u^{-1} \sum_{p} \left(\frac{\sin^2\theta_d \sin^2\theta_u (f_u - f_d)}{E_{1u} - E_{1d} - \omega} + \frac{f_u \cos^2\theta_d \sin^2\theta_u}{E_{1u} - E_{2d} - \omega} - \frac{f_d \cos^2\theta_u \sin^2\theta_d}{E_{2u} - E_{1d} - \omega} \right) \\
V &= 2G_0 N_u^{-1} \sum_{p} \left(\frac{\cos^2\theta_d \cos^2\theta_u (f_u - f_d)}{E_{1u} - E_{1d} - \omega} + \frac{f_u \cos^2\theta_u \sin^2\theta_d}{E_{1u} - E_{2d} - \omega} - \frac{f_d \cos^2\theta_d \sin^2\theta_u}{E_{2u} - E_{1d} - \omega} \right).
\end{aligned}
\tag{4.11}
$$

Here the indices d and u replace the pair of indices $\boldsymbol{p}$ and α. That is, we have $u \equiv (\boldsymbol{p} + \boldsymbol{q}/2, \uparrow)$, $d \equiv (\boldsymbol{p} - \boldsymbol{q}/2, \downarrow)$ and $f_{u(d)} \equiv f(E_{1u(d)})$. The set of equations (4.10) enables us to investigate different problems such as collective spin-wave excitations or the response of the system to transverse magnetic fields. The latter problem will be discussed in the next section.

First we consider collective spin-wave excitations in zero magnetic field, i.e. $H_0 = H_1 = 0$. For long-wave excitations when $q \ll k_F$ and $\boldsymbol{q}\nabla_{\boldsymbol{k}}\varepsilon(\boldsymbol{k}) \ll T_0$ in the leading order in q the coefficients P, Q, ∇ and R have the simple forms

$$
\begin{aligned}
P = R &= 2G_0\rho_0(\sin^2\theta_F \Gamma(q,\omega) + A(\omega)) \\
V &= 2G_0\rho_0(\cos^2\theta_F \Gamma(q,\omega) - A(\omega)) \\
Q &= 2G_0\rho_0(\sin^2\theta_F \tan^2\theta_F \Gamma(q,\omega) - A(\omega))
\end{aligned}
\tag{4.12}
$$

where $A(\omega)$ and $\Gamma(q,\omega)$ are determined by equations (3.7) and (3.10), respectively. The spectrum of spin-wave excitations is determined by the condition that the determinant of the set of equations (4.10) is equal to zero:

$$
(P-1)(R-1) = VQ. \tag{4.13}
$$

First we consider low-frequency waves with $\omega \ll T_0$ when $A(\omega) \approx A(0)$. Substituting (4.12) into (4.13) leads to equation (3.20) with the negative Landau parameter $F_0 = -T_m/T_0$. This parameter is the usual antisymmetric Landau parameter F_0^a, i.e. $F_0^a = -T_m/T_0$. Indeed, the static spin susceptibility is related to F_0^a by the conventional relation

$$
\chi = \chi_0 \frac{m^*/m_0}{1 + F_0^a}.
$$

Substituting $F_0^a = -T_m/T_0$ gives the result in [11]. In the paramagnetic heavy-fermion state we have $T_m < T_0$ and $-1 < F_0^a < 0$. In this case, according to the detailed analysis given in [12], equation (3.20) has only a complex root. Such a solution describes spin-wave excitations that decay owing to the Landau decay. The negative parameter F_0^a gives evidence for an attraction between heavy quasi-particles that can stimulate the superconducting coupling. Note that this result agrees with Anderson's speculation [16] that, in heavy-fermion superconductors triplet, pairing can take place.

Apart from the damped spin waves discussed above, equation (4.13) has a solution that corresponds to optical-like spin waves with a non-zero frequency $\omega < 2b$ at $q = 0$. To find this solution, one can note that at $q = 0$ we have $\Gamma(q = 0, \omega) = 0$. Then according to (4.12) the parameters P, Q, V and R are determined completely by the function $A(\omega)$ given by (3.7). Using (3.16) and (3.17), one obtains

$$A(\omega) = 4b^2 \int_{\theta_0}^{\theta_F} \frac{\sin(2\theta)\,\mathrm{d}\theta}{4b^2 - \omega^2 \sin^2(2\theta)} = \frac{x^2}{(x^2-1)^{1/2}} \tan^{-1}\left(\frac{1}{(x^2-1)^{1/2}}\right) \tag{4.14}$$

where $x = 2b/\omega > 1$, and we have used the facts that $\theta_0 \approx 0$ and $2\theta_F \approx \pi$. Substituting (4.12) with $\Gamma(q = 0, \omega) = 0$ into (4.13) gives the spectral equation

$$A(\omega) = 1/4G_0\rho_0. \tag{4.15}$$

We shall suppose that the exchange interaction parameter G_0 is much smaller than the conduction band width, i.e. $G_0\rho_0 \ll 1$. Then equation (4.15) has the following solution:

$$\omega_0 = 2b(1 + 4\pi^2 G_0^2 \rho_0^2)^{-1/2} = 2T_0(m^*/m_0 - \pi^2 T_m/T_0)^{1/2}. \tag{4.16}$$

Note that ω_0 is smaller than the minimum direct gap between the hybridized bands (3.4). Hence, at least at small q ($qv_F \ll 2b$) the attenuation of the spin waves owing to interband electron transitions is forbidden by the energy conservation law. The same situation occurs for intraband transitions. From the mathematical point of view this is related to the analyticity of the functions $A(\omega)$ and $\Gamma(q, \omega)$ at small q in the neighbourhood of the point $\omega = \omega_0$. Thus, only the spin waves with the gap ω_0 are stable in the paramagnetic heavy-fermion state.

It should be emphasized that the exchange interaction with the parameter G_0 and the RKKY interaction play decisive roles in the collective spin-wave excitations. At $G_0 = 0$ there are no collective spin-wave excitations because equation (4.13) has no solution.

5. Electron spin resonance

Now we consider the case of non-zero external field H_0. We shall study the response of the system on a non-uniform transverse magnetic field $H_1(\boldsymbol{r}, t)$ with frequency ω. According to sections 2 and 4 the transverse field changes the distribution function $N = N_0 + N_1$ and gives rise to a local transverse magnetic moment $\boldsymbol{M}_t(\boldsymbol{r}, t)$ that is the sum of moments of conduction and f electrons:

$$\begin{aligned}\boldsymbol{M}_t(\boldsymbol{r}, t) = \boldsymbol{M}_c(\boldsymbol{r}, t) + \boldsymbol{M}_f(\boldsymbol{r}, t) &= \tfrac{1}{2} g_c \mu_B N_u^{-1} \sum_{\gamma\delta\boldsymbol{p}} \boldsymbol{\sigma}_{\gamma\delta} N^c_{1\delta\gamma}(\boldsymbol{p}, \boldsymbol{r}, t) \\ &\quad + \tfrac{1}{2} g_f \mu_B N_u^{-1} \sum_{\gamma\delta\boldsymbol{p}} \boldsymbol{\sigma}_{\gamma\delta} N^f_{1\delta\gamma}(\boldsymbol{p}, \boldsymbol{r}, t).\end{aligned} \tag{5.1}$$

In the isotropic case when the function $N_1(\boldsymbol{p}, \boldsymbol{r}, t)$ does not depend on the direction of the wavenumber $\boldsymbol{p}$, the local moment (5.1) may be related to the local transverse effective fields $\boldsymbol{h}^c$ and $\boldsymbol{h}^f$ that are determined by (2.7). Comparing (2.7) and (5.1) we find that

$$\tfrac{1}{2} g_f \mu_B \boldsymbol{h}^c(\boldsymbol{r}, t) + \tfrac{1}{2} g_c \mu_B \boldsymbol{h}^f(\boldsymbol{r}, t) = \tfrac{1}{2} g_c g_f \mu_B^2 \boldsymbol{H}_1(\boldsymbol{r}, t) - G_0 \boldsymbol{M}_t(\boldsymbol{r}, t). \tag{5.2}$$

Using the variables (4.6), for the time and space periodic transverse field

$$\boldsymbol{H}_1(\boldsymbol{r}, t) = \boldsymbol{H}_1 \exp(\mathrm{i}\boldsymbol{q}\cdot\boldsymbol{r} - \mathrm{i}\omega t) + \mathrm{HC} \tag{5.3}$$

equation (5.2) takes the form

$$\tfrac{1}{2} g_f \mu_B h^c_+(q, \omega) + \tfrac{1}{2} g_c \mu_B h^f_+(q, \omega) = \tfrac{1}{2} g_c g_f \mu_B^2 H_{1+} - G_0 M_{t+}(q, \omega). \tag{5.4}$$

The fields h_+^c and h_+^f may be found from equations (4.10). One obtains

$$
\begin{aligned}
h_+^f(q,\omega) &= \tfrac{1}{2}\mu_B H_{1+}\frac{g_c V + g_f(1-P)}{(R-1)(P-1)-QV} \\
h_+^c(q,\omega) &= \tfrac{1}{2}\mu_B H_{1+}\frac{g_f Q + g_c(1-R)}{(R-1)(P-1)-QV}.
\end{aligned}
\tag{5.5}
$$

In the resonance case the denominator becomes equal to zero. Therefore the resonance frequency is determined by equation (4.13). A general solution of the equation at $H_0 \neq 0$ is too complicated. In addition to the spin waves studied in section 4 we consider the case of the uniform field $\boldsymbol{H}_1$ with $q = 0$. Then the parameters P, Q, V and R are given by (4.12) with

$$
\Gamma(q=0,\omega) = \frac{2h_0}{2h_0-\omega} \tag{5.6}
$$

where

$$
h_0 = h_0^c \cos^2\theta_F + h_0^f \sin^2\theta_F = \tfrac{1}{2} g_f \mu_B H_0 (1 - T_m/T_0)^{-1}. \tag{5.7}
$$

This equality follows from (3.19) and (4.5). At low frequencies $\omega \ll T_0$ we can suppose that $A(\omega) \approx A(0)$ with $A(0)$ given by (3.18). Then equation (4.13) for the resonance frequency takes the form

$$
\Gamma(q=0,\omega) = -T_0/T_m. \tag{5.8}
$$

Using (5.6), it is easy to find the resonance frequency

$$
\omega_r = 2h_0(1 - T_m/T_0) = g_f \mu_B H_0. \tag{5.9}
$$

Thus we obtain well known results of the Landau Fermi-liquid theory for normal metals. Namely, the frequency of the electron spin resonance does not depend on the specific properties of the systems under consideration. However, it is important to draw attention to the following fact. Although the system consists of conduction and f electrons having spin $\frac{1}{2}$ and the gyromagnetic factors g_c and g_f, respectively, the main contribution to electron spin resonance is given by f electrons.

6. Conclusions and discussion

In the present paper we have applied the Landau Fermi-liquid theory proposed in [11] for describing dynamic phenomena in heavy-fermion compounds at temperatures below the Kondo temperature T_0 when the systems are in the coherent heavy-fermion state. We have derived the kinetic equation and have solved it in the case when collisions of quasi-particles are negligible. Within the phenomenological approach, collective zero-sound and spin-wave excitations have been studied. Although we have considered only symmetric modes, the approach enables us to investigate other modes.

Neglecting the Coulomb interaction, we have found that there are no zero-mode excitations in the system under consideration. This result contradicts [6] where within the lattice Anderson model via the slave-boson $1/N$ expansion the zero-sound modes have been found. To understand the reason for the discrepancy we have considered an approximation within which the phase fluctuations of the coherence parameter are decoupled from fluctuations of the coherence parameter magnitude and the effective f-level energy. This approximation gives results in the complete agreement with [6]. It enables us to suppose that, within the slave-boson approach, complete consideration of the interactions between different fluctuations is necessary, going beyond the next to leading order in $1/N$.

However, at the present time there is no method that enables us to sum diagrams of $O(1/N^2)$ and higher orders. Unlike the slave-boson approach, our approach is not based on the large-orbital degeneracy expansion and takes into account a certain interaction between the fluctuations.

The competition between the magnetic RKKY interaction and the coherent Kondo effect is one of the problems that may be studied using the Landau Fermi-liquid theory. In the previous paper [11] we have studied the stability of the coherent heavy-fermion state with respect to ferromagnetism. The results obtained here enable us to estimate the effect of the RKKY interaction on collective spin waves. We have found that it is the ratio of the RKKY interaction scale T_m to the Kondo scale T_0 which determines the spectrum of collective spin-wave modes. Acoustic-like spin waves are determined by the conventional equations (3.20) with the negative Landau parameter $F_0^a = -T_m/T_0$. In the paramagnetic heavy-fermion state when $T_m < T_0$ these waves are decaying owing to the Landau decay. Moreover the negative F_0^a indicates an attraction between heavy quasi-particles. There are also stable optical-like spin waves. At $q = 0$ their frequency (4.16) is slightly lower than the minimum direct energy gap between hybridized quasi-particle bands.

We have also found the symmetrical Landau parameter $F_0^s = m^*/m_0 - 1$ in accordance with the microscopic approaches [14, 15]. In agreement with the result at low temperatures the compressibility (3.23) of the heavy-fermion system tends to the non-renormalized value.

We have also studied the response of the system in an external magnetic field H_0 with an alternative transverse magnetic field. It has been found that the frequency (5.9) of electron spin resonance is mainly determined by f electrons and does not depend on the specific properties of the system. We believe that an investigation of the electron spin resonance in heavy-fermion compounds at temperatures $T < T_0$ enables us to obtain very important information on the magnetic state of the f ions.

Appendix 1

In this appendix we present the approximate solution (2.23) of the linearized kinetic equation (2.13). In the paramagnetic phase the substitution of the matrix ε_1 given by equations (3.1) and (3.5) into equation (2.23) leads to the following results for the matrix elements of the fluctuation matrix N_1:

$$
\begin{aligned}
N_1^c(\boldsymbol{q},\omega) = \varepsilon_1^f & \left[\frac{(\boldsymbol{q}\cdot\boldsymbol{v}_{\boldsymbol{k}})\sin^2\theta_{\boldsymbol{k}}\cos^2\theta_{\boldsymbol{k}}}{(\boldsymbol{q}\cdot\boldsymbol{v}_{\boldsymbol{k}})-\omega} f'(E_{1\boldsymbol{k}}) + \frac{f(E_{1\boldsymbol{k}})\Delta_{\boldsymbol{k}}\sin^2(2\theta_{\boldsymbol{k}})}{2(\Delta_{\boldsymbol{k}}^2-\omega^2)} \right] \\
& + \varepsilon_1^{cf}\bigg[-\frac{(\boldsymbol{q}\cdot\boldsymbol{v}_{\boldsymbol{k}})\sin\theta_{\boldsymbol{k}}\cos^3\theta_{\boldsymbol{k}}}{(\boldsymbol{q}\cdot\boldsymbol{v}_{\boldsymbol{k}})-\omega} f'(E_{1\boldsymbol{k}}) \\
& - \tfrac{1}{2} f(E_{1\boldsymbol{k}})\sin(2\theta_{\boldsymbol{k}})\left(\frac{\cos^2\theta_{\boldsymbol{k}}}{\Delta_{\boldsymbol{k}}-\omega} - \frac{\sin^2\theta_{\boldsymbol{k}}}{\Delta_{\boldsymbol{k}}+\omega}\right)\bigg] \\
& + \varepsilon_1^{fc}\bigg[-\frac{(\boldsymbol{q}\cdot\boldsymbol{v}_{\boldsymbol{k}})\sin\theta_{\boldsymbol{k}}\cos^3\theta_{\boldsymbol{k}}}{(\boldsymbol{q}\cdot\boldsymbol{v}_{\boldsymbol{k}})-\omega} f'(E_{1\boldsymbol{k}}) \\
& - \tfrac{1}{2} f(E_{1\boldsymbol{k}})\sin(2\theta_{\boldsymbol{k}})\left(\frac{\cos^2\theta_{\boldsymbol{k}}}{\Delta_{\boldsymbol{k}}+\omega} - \frac{\sin^2\theta_{\boldsymbol{k}}}{\Delta_{\boldsymbol{k}}-\omega}\right)\bigg] \qquad \text{(A1.1)} \\
N_1^f(\boldsymbol{q},\omega) = \varepsilon_1^f & \left[\frac{(\boldsymbol{q}\cdot\boldsymbol{v}_{\boldsymbol{k}})\sin^4\theta_{\boldsymbol{k}}}{(\boldsymbol{q}\cdot\boldsymbol{v}_{\boldsymbol{k}})-\omega} f'(E_{1\boldsymbol{k}}) - \frac{f(E_{1\boldsymbol{k}})\Delta_{\boldsymbol{k}}\sin^2(2\theta_{\boldsymbol{k}})}{2(\Delta_{\boldsymbol{k}}^2-\omega^2)} \right]
\end{aligned}
$$

$$+\varepsilon_1^{cf}\bigg[-\frac{(\boldsymbol{q}\cdot\boldsymbol{v_k})\sin^3\theta_{\boldsymbol{k}}\cos\theta_{\boldsymbol{k}}}{(\boldsymbol{q}\cdot\boldsymbol{v_k})-\omega}f'(E_{1\boldsymbol{k}})$$
$$+\tfrac{1}{2}f(E_{1\boldsymbol{k}})\sin(2\theta_{\boldsymbol{k}})\left(\frac{\cos^2\theta_{\boldsymbol{k}}}{\Delta_{\boldsymbol{k}}-\omega}-\frac{\sin^2\theta_{\boldsymbol{k}}}{\Delta_{\boldsymbol{k}}+\omega}\right)\bigg]$$
$$+\varepsilon_1^{fc}\bigg[-\frac{(\boldsymbol{q}\cdot\boldsymbol{v_k})\sin^3\theta_{\boldsymbol{k}}\cos\theta_{\boldsymbol{k}}}{(\boldsymbol{q}\cdot\boldsymbol{v_k})-\omega}f'(E_{1\boldsymbol{k}})$$
$$+\tfrac{1}{2}f(E_{1\boldsymbol{k}})\sin(2\theta_{\boldsymbol{k}})\left(\frac{\cos^2\theta_{\boldsymbol{k}}}{\Delta_{\boldsymbol{k}}+\omega}-\frac{\sin^2\theta_{\boldsymbol{k}}}{\Delta_{\boldsymbol{k}}-\omega}\right)\bigg] \qquad \text{(A1.2)}$$

$$N_1^{cf}(\boldsymbol{q},\omega)=N_1^{fc}(-\boldsymbol{q},-\omega)=\varepsilon_1^{f}\bigg[-\frac{(\boldsymbol{q}\cdot\boldsymbol{v_k})\sin^3\theta_{\boldsymbol{k}}\cos\theta_{\boldsymbol{k}}}{(\boldsymbol{q}\cdot\boldsymbol{v_k})-\omega}f'(E_{1\boldsymbol{k}})$$
$$+\tfrac{1}{2}f(E_{1\boldsymbol{k}})\sin(2\theta_{\boldsymbol{k}})\left(\frac{\cos^2\theta_{\boldsymbol{k}}}{\Delta_{\boldsymbol{k}}-\omega}-\frac{\sin^2\theta_{\boldsymbol{k}}}{\Delta_{\boldsymbol{k}}+\omega}\right)\bigg]$$
$$+\varepsilon_1^{cf}\left[\frac{(\boldsymbol{q}\cdot\boldsymbol{v_k})\sin^2\theta_{\boldsymbol{k}}\cos^2\theta_{\boldsymbol{k}}}{(\boldsymbol{q}\cdot\boldsymbol{v_k})-\omega}f'(E_{1\boldsymbol{k}})-f(E_{1\boldsymbol{k}})\left(\frac{\cos^4\theta_{\boldsymbol{k}}}{\Delta_{\boldsymbol{k}}-\omega}+\frac{\sin^4\theta_{\boldsymbol{k}}}{\Delta_{\boldsymbol{k}}+\omega}\right)\right]$$
$$+\varepsilon_1^{fc}\left[\frac{(\boldsymbol{q}\cdot\boldsymbol{v_k})\sin^2\theta_{\boldsymbol{k}}\cos^2\theta_{\boldsymbol{k}}}{(\boldsymbol{q}\cdot\boldsymbol{v_k})-\omega}f'(E_{1\boldsymbol{k}})+\frac{f(E_{1\boldsymbol{k}})\Delta_{\boldsymbol{k}}\sin^2(2\theta_{\boldsymbol{k}})}{2(\Delta_{\boldsymbol{k}}^2-\omega^2)}\right] \qquad \text{(A1.3)}$$

where for brevity we denote $\varepsilon_1^{ab}(\boldsymbol{q},\omega)\equiv\varepsilon_1^{ab}$, $f'(x)\equiv \mathrm{d}f(x)/\mathrm{d}x$. Deriving these equations, we have used that

$$E_{1\boldsymbol{k}+\boldsymbol{q}/2}-E_{1\boldsymbol{k}-\boldsymbol{q}/2}\approx\boldsymbol{q}\cdot\nabla_{\boldsymbol{k}}E_{1\boldsymbol{k}}\equiv(\boldsymbol{q}\cdot\boldsymbol{v_k})$$
$$f(E_{1\boldsymbol{k}+\boldsymbol{q}/2})-f(E_{1\boldsymbol{k}-\boldsymbol{q}/2})\approx(\boldsymbol{q}\cdot\boldsymbol{v_k})f'(E_{1\boldsymbol{k}}) \qquad \text{(A1.4)}$$
$$E_{2\boldsymbol{k}+\boldsymbol{q}/2}-E_{1\boldsymbol{k}-\boldsymbol{q}/2}\approx E_{2\boldsymbol{k}}-E_{1\boldsymbol{k}}+\boldsymbol{q}\cdot\nabla_{\boldsymbol{k}}\varepsilon(\boldsymbol{k})\approx E_{2\boldsymbol{k}}-E_{1\boldsymbol{k}}\equiv\Delta_{\boldsymbol{k}}$$

where $\boldsymbol{v_k}=\nabla_{\boldsymbol{k}}E_{1\boldsymbol{k}}$ is the velocity of heavy quasi-particles in the band $E_{1\boldsymbol{k}}\cdot\boldsymbol{v_k}$ is related to the electron velocity $\boldsymbol{v}_{0\boldsymbol{k}}=\nabla_{\boldsymbol{k}}\varepsilon(\boldsymbol{k})$ in the conduction band $\varepsilon(\boldsymbol{k})$ by the equation

$$\boldsymbol{v_k}=\nabla_{\boldsymbol{k}}E_{1\boldsymbol{k}}=(\partial E_{1\boldsymbol{k}}/\partial\varepsilon(\boldsymbol{k}))\nabla_{\boldsymbol{k}}\varepsilon(\boldsymbol{k})=\boldsymbol{v}_{0\boldsymbol{k}}\cos^2\theta_{\boldsymbol{k}}. \qquad \text{(A1.5)}$$

$\Delta_{\boldsymbol{k}}$ is the direct energy gap between the lower band $E_{1\boldsymbol{k}}$ and upper band $E_{2\boldsymbol{k}}$. The latter equation in (A1.4) results from the fact that minimum value of $\Delta_{\boldsymbol{k}}$ is equal to $2b\approx(\mu T_0)^{1/2}\gg T_0$.

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